

# **An Introduction to the Sources of Delivery Error for Direct-Fire Ballistic Projectiles**

**by Luke S. Strohm**

**ARL-TR-6494**

**July 2013**

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**ARL-TR-6494****July 2013**

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## **An Introduction to the Sources of Delivery Error for Direct-Fire Ballistic Projectiles**

**Luke S. Strohm**

**Weapons and Materials Research Directorate, ARL**

| REPORT DOCUMENTATION PAGE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                             |                              | Form Approved<br>OMB No. 0704-0188                         |                                                        |                                                           |
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| 14. ABSTRACT<br><p>This report describes the primary sources of delivery error for direct-fire ballistic projectiles. I explain how error sources and their resulting shot distributions are characterized statistically. Next, I describe the major types of error sources and the steps a shooter might take to reduce them. Three major categories of error sources are outlined: fixed bias errors, variable bias errors, and random errors. For a weapon system, fixed bias errors always remain constant, while random errors always change. Variable bias errors stay constant within an engagement but change between engagements. The report concludes with detailed descriptions of error sources within each category.</p>                                                                                                                                                                                                                               |                             |                              |                                                            |                                                        |                                                           |
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# 1. Introduction

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The purpose of this report is to describe the primary sources of delivery error for direct-fire ballistic projectiles. While there is much overlap in the error sources affecting different types of direct-fire weapons, this report focuses on small-caliber weapon systems\* and does not cover some of the error sources associated with tanks or other large platforms (primarily related to their sophisticated fire control systems).

The term “delivery error,” also referred to as “gunnery error” (3, 4), is a general term referring to the error in delivering a projectile upon a target (5–7). The terms “delivery accuracy” or simply “accuracy” have also been used with a similar meaning (8, 9), although some researchers have provided a more exact definition of this term.<sup>†</sup> Because of the generality in the way these two terms are used, researchers have relied on additional statistical terms to quantify system performance (8), which we shall discuss in section 2 of this report. In addition, we will discuss how these statistical measures can be used to calculate the probability of hit ( $P_h$ )—one of the measures of a weapon system’s effectiveness against a target.

Besides understanding how delivery error is quantified, this report discusses the many error sources that contribute to delivery error. Bornstein et al. (12) refer to them as a “hierarchy” of error sources, which together form an error budget. Different terminology and definitions have been attributed by researchers to many of the error sources, which we will also discuss throughout the report. In addition, error budgets differ in their level of detail—the error components have been continually divided into smaller components as researchers grow in their understanding of what is fundamentally causing the errors. Understanding where error sources fit together in an ever-expanding hierarchy is important to avoid double-counting and attain a proper system analysis.

I have three goals for this report:

1. Explain how error sources and their resulting shot distributions are characterized statistically (section 2).
2. Describe the major types of error sources and the steps a shooter might take to reduce them (section 3).

---

\* Joint Publication 1-02 defines a weapon system as a “combination of one or more weapons with all related equipment, materials, services, personnel, and means of delivery and deployment (if applicable) required for self-sufficiency” (1). Often researchers have referred to a weapon system as a weapon, round, and fire control system, excluding the human component (2). For small-caliber weapons (as opposed to some tank systems), the error sources associated with the user are always an integral part of an analysis of delivery error, whether the human component is formally included in the definition of weapon system or not.

<sup>†</sup> For example, delivery accuracy has been defined as a shot’s “miss distance” measured in the target plane between the desired impact point and the actual impact point (10), while others define delivery accuracy by the angle subtended by this miss distance at a given range (11).

3. Provide detailed descriptions of error sources within each type (sections 4–6). Each section describes the error sources that are often most significant first, followed by other error sources.

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## 2. Statistical Characterization of Error Sources and Target Impact Distributions

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### 2.1 Accuracy and Precision

Accuracy and precision, often misunderstood and misapplied, are concepts that are critical for understanding and discussing error sources and resulting target impact distributions. *Accuracy* is defined as “the ability to measure the true value of the characteristic correctly on average,” while *precision* is defined as “the inherent variability in the measurements” (13). While accuracy and precision are formally defined with regard to measurement systems, the terms can also be used to characterize general data sets, such as the shot impacts produced by a weapon system (3, 7, 14, 15). In layman’s terms, accuracy is how close the average data point is to the correct point (or desired point, i.e., the target bullseye), while precision is how close the data points are to each other, with a high precision indicating a small spread.

Consider figure 1 (16), which illustrates how accuracy and precision apply to a target impact distribution. The average impact location of a shot distribution is typically referred to as the center of impact (COI) or alternatively the mean point of impact (MPI) (1). Distribution A has its COI close to the bullseye, showing a high accuracy, but the shots are far apart from each other, showing low precision. Distribution B has its COI far from the bullseye, showing low accuracy, but its shots are in close proximity to each other, showing high precision.

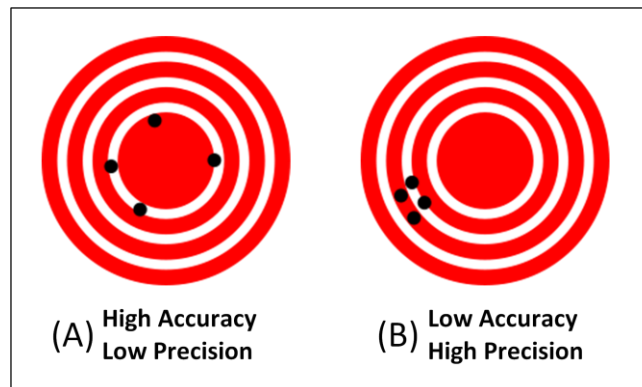


Figure 1. Accuracy vs. precision for target impacts (16).

As a preview of topics to be discussed later, the accuracy of a target impact distribution as defined here equals the *bias* of the distribution, while the precision is equal to the (total) *dispersion*\* of the distribution (also referred to as “target impact dispersion” [TID]) (17). Both accuracy and precision are important for attaining a high  $P_h$  on the bullseye. However, while one can occasionally hit the bullseye with high accuracy and low precision (A), the combination of low accuracy and high precision (B) virtually precludes any hits on the bullseye. This is the plight of the sniper, who has high precision, but can still miss the bullseye because of bias errors (7).

As stated in the beginning of this section, not all researchers have used accuracy and precision in the way discussed here, often interchanging the two terms loosely (see Weaver [7] for an extended discussion, which includes several examples). Thus, care must be taken when understanding what one means by these terms. Fortunately, researchers typically use additional terms to quantify accuracy and precision, which are often used more consistently, thus avoiding confusion. We shall discuss these terms in the next few sections.

## 2.2 Normal (Gaussian) Distributions

For direct-fire ballistic projectiles, it is common to assume that error sources and the shot distributions they produce can be characterized by normal (Gaussian) distributions. Normal distributions are defined by a mean ( $\mu$ ) and standard deviation ( $\sigma$ , SD), which produce a bell curve that is unique to the distribution. The mean is the average of the distribution, while the SD quantifies the spread or precision of the distribution. For a one-dimensional normal distribution, approximately 68% of the distribution is within one SD of the mean (+/-) and 95% within two SDs (figure 2).

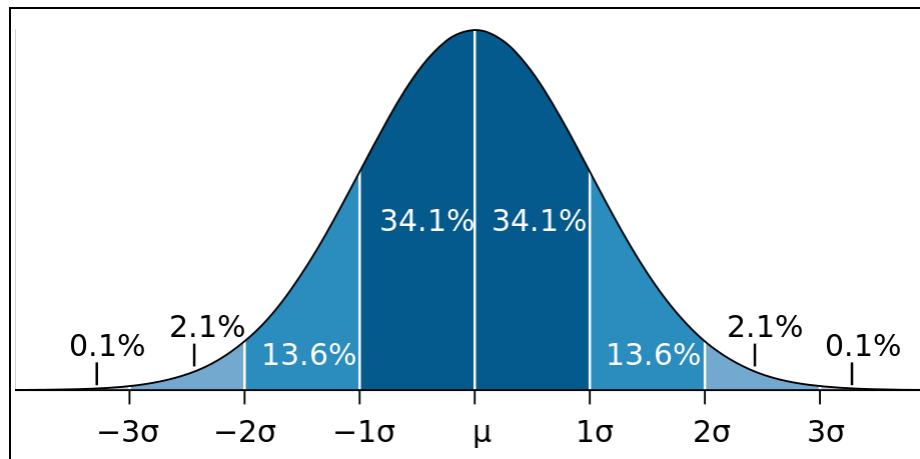


Figure 2. Percentage of area under portions of a bell curve (18).

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\**Dispersion* is defined in the Joint Publication 1-02 as “a scattered pattern of hits around the mean point of impact of bombs and projectiles dropped or fired under identical conditions” (1). Statistically, the term is used to describe the precision of the total round-to-round variation. This term is sometimes used to refer to the ammunition dispersion (see discussion in section 6.2).

In two dimensions, target impact distributions follow a bivariate normal distribution, meaning that the impact locations vary normally in two directions—in this case the horizontal and vertical directions of the target plane. Referring to figure 3, the distribution's mean ( $\mu_x, \mu_y$ ) is equal to the COI. The distance between the COI and the desired point of impact (DPI, often referred to as the aim point\* or point of aim) is the distribution's bias (for an individual shot, the distance between point of impact [POI] and DPI is often referred to as a shot's miss distance, which we shall use as a term throughout this report). It is common to represent the distribution's horizontal and vertical SDs by drawing a  $1\sigma$  probability contour that takes on the shape of an ellipse with semi-minor and semi-major axes equal to  $1\sigma$  in each direction,  $\sigma_x$  and  $\sigma_y$ . The probability density function (PDF), which quantifies the likelihood of a random variable (shot impacts) occurring, is equal at all points along the probability contour. Throughout this report, we will represent a distribution's SDs in this manner. Error sources that are constant bias the distribution, moving the COI ( $\mu_x, \mu_y$ ) away from the desired impact point. Error sources that vary change the distribution's SDs ( $\sigma_x, \sigma_y$ ).

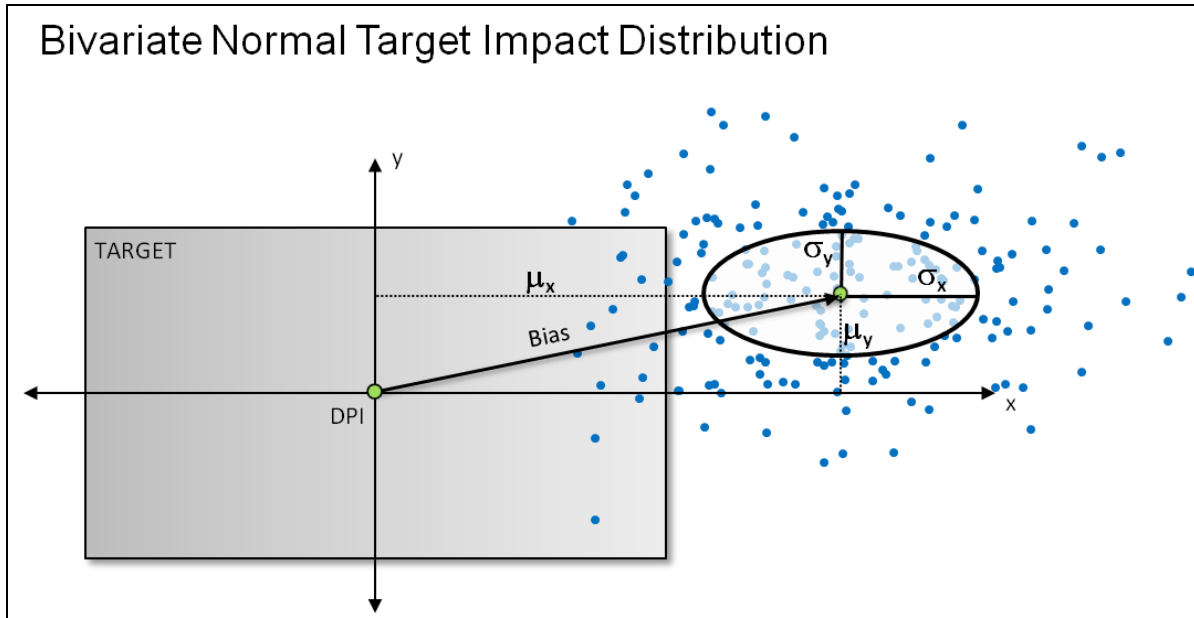


Figure 3. Bivariate normal target impact distribution.

### 2.3 Percentage of Impacts Occurring Within a Region

To determine the percentage of a distribution that is within a given region, one must integrate the PDF for a bivariate normal distribution over that region. Assuming no correlation between  $\sigma_x$  and  $\sigma_y$  (which we will assume for all cases in this discussion), equation 1 shows the PDF integrated over a rectangular region bounded by points  $x_1, x_2, y_1$ , and  $y_2$ .

---

\* Researchers are sometimes hesitant to use the term aim point, because there can be confusion over whether it refers to where the barrel is pointing at on the target upon firing (19) or whether it refers to the DPI.

$$\%_{Rectangle} = \iint_{x_1, y_1}^{x_2, y_2} \frac{\exp \left[ -\frac{1}{2} \left[ \left( \frac{x - \mu_x}{\sigma_x} \right)^2 + \left( \frac{y - \mu_y}{\sigma_y} \right)^2 \right] \right]}{2\pi\sigma_x\sigma_y} dy dx . \quad (1)$$

If the rectangular region is centered on the COI and is bounded by  $\mu \pm 1\sigma$ ,  $2\sigma$ , and  $3\sigma$  along each axis, we arrive at figure 4. Using equation 1, we determine that 47%, 91%, and 99% of impacts statistically occur within concentric  $1\sigma$ ,  $2\sigma$ , and  $3\sigma$  rectangles, respectively (figure 4). A slightly more complicated integration is required to determine the percentage of impacts within an ellipse as discussed in the previous section. Upon integration, 39%, 86%, and 99% of impacts are likely to occur within  $1\sigma$ ,  $2\sigma$ , and  $3\sigma$  ellipses, respectively (figure 5).

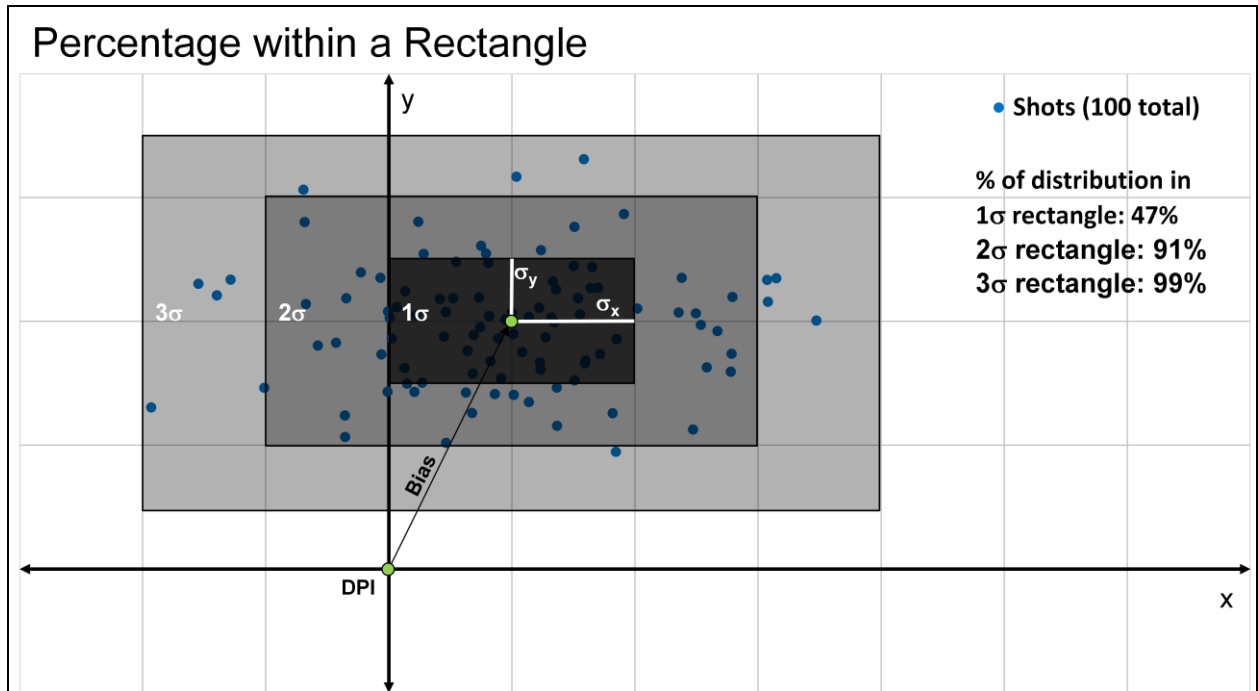


Figure 4. Percentage of bivariate normal distribution within a rectangle.

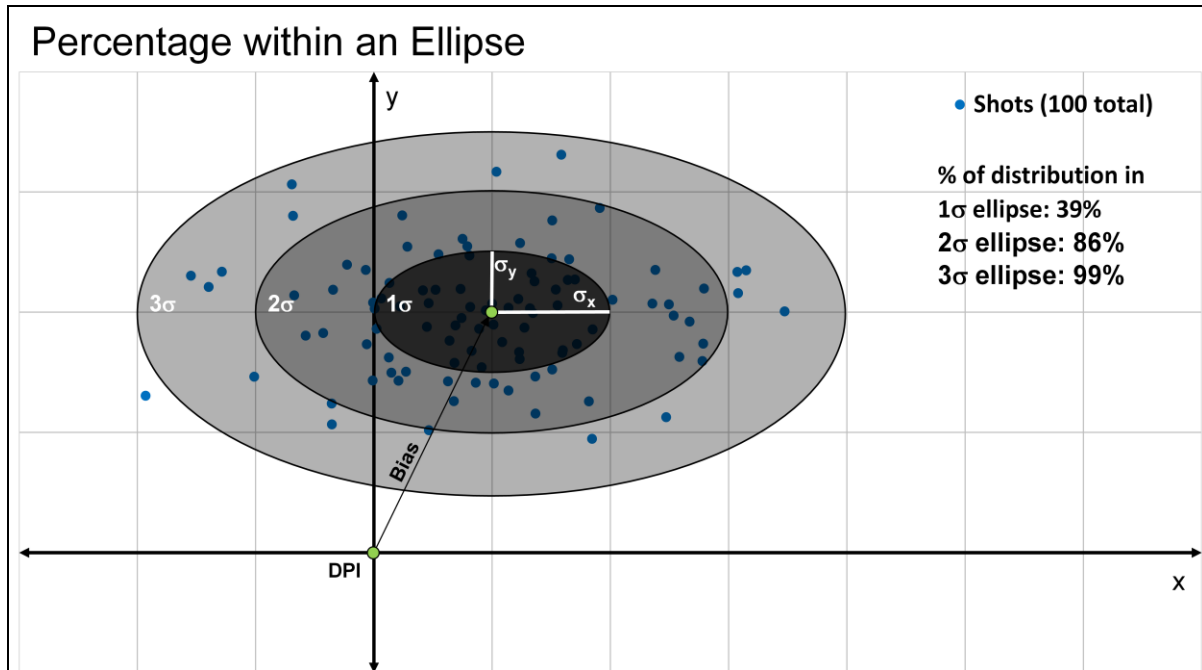


Figure 5. Percentage of bivariate normal distribution within an ellipse.

For more complex target shapes and/or shot distributions, researchers have often relied on trajectory models in Monte Carlo simulation to determine the  $P_h$  against the target (10, 20). The Monte Carlo simulation involves running the trajectory model a large number of times, or iterations, and changing uncertain variables (the error sources) in each trajectory iteration through random draw. Thus, over a large number of iterations, the simulation captures the variations in performance that are statistically likely to occur. For each iteration, if the shot impact is within the target bounds, it is recorded as a hit. Thus, at the end of the simulation, if there were 500 hits recorded out of 10,000 iterations, the  $P_h$  would be the ratio of the two numbers, or 0.05.

## 2.4 Other Ways of Describing Precision

The precision of a target impact distribution is sometimes quantified using different measures than the SD. Grubbs (8) details many of the alternative ways to measure precision, of which we shall examine three:

- **Circular Error Probable (CEP)\***: The CEP specifies the radius of the circle measured in the target plane and centered on the COI that contains 50% of the round impacts.<sup>†</sup> If one assumes a bivariate normal distribution of round impacts, the CEP may be converted to an equivalent SD. By performing an integration similar to those in section 2.3, one finds that the CEP equals  $1.18\sigma$ .

\* Or Circular Probable Error (CPE).

<sup>†</sup> Grubbs states that the CEP can also be measured for a circle centered on the DPI, as opposed to the COI. This usage, however, has not been observed in more recent literature.

- **Probable Error (PE):** While the CEP describes precision in two dimensions, the PE only quantifies precision in one dimension. The PE specifies the interval about both sides of the mean that contains 50% of the shots. If one assumes a normal distribution, the PE is equivalent to  $0.67\sigma$ .
- **Extreme Spread:** The extreme spread is defined as the maximum distance measured in the target plane between all possible pairs of shot impacts. This is a simple method used to characterize precision quickly when taking manual measurements of target impacts (21).

## 2.5 Unit Effects

Researchers have often sought to establish “unit effects” to correlate error source and target miss distance (2, 5, 11, 22, 23). A unit effect quantifies how many units of miss distance are caused by one unit of error source. For many direct-fire ballistic applications, this is a useful approximation, which assumes a linear relationship between error source and miss distance and typically assumes that error sources are uncorrelated (meaning they do not affect each other). If individual error sources are indeed assumed to be uncorrelated, their separate contributions to the delivery error may be easily combined. Biases are simply added to determine the COI, and SDs are combined as the square root of the sum of the squares of the individual SDs (“root-sum-square” or RSS):

$$\sigma_{total} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \dots} . \quad (2)$$

Even though it is common and often reasonable to assume uncorrelated error sources, some error sources are indeed correlated. For example, muzzle velocity variation and wind drift are correlated. This is because changes in muzzle velocity affect projectile time of flight, which affects the distance a projectile drifts due to the wind. For a more exact calculation of the miss distance caused by correlated error sources, researchers have often used trajectory models (10).

Often error sources or the precision of target impact distributions are specified by angles. Because of their small size, angles are usually expressed in units other than degrees:

- **Minutes of Angle (MOA):** 1 MOA equals 1/60 of a degree ( $0.017^\circ$ ). MOAs are also referred to as “minutes of arc” or “arcminutes.” This measure is often used to quantify the extreme spread of a target distribution at a given range or to adjust a weapon’s sights (24–26).
- **Milliradians (mrad):**  $2\pi$  radians equal  $360^\circ$ , the angle subtended by a circle. 1 mrad is equal to 1/1000 of a radian and is therefore approximately  $0.057^\circ$ .
- **Mils:** 1 mil equals 1/6400 of the angle subtended by a circle and is approximate to a mrad:

$$1 \text{ mil} = \left( \frac{1}{6400} \text{ circle} \right) \cdot \frac{2\pi \text{ rad}}{\text{circle}} \cdot \frac{1000 \text{ mrad}}{\text{rad}} = 0.98 \text{ mrad} . \quad (3)$$

When direct-fire delivery error is discussed, these angular units are often used because many error sources produce an angular adjustment to the initial trajectory angle. For small angles, we may approximate the miss distance caused by the angular trajectory adjustment as

$$\text{Miss Distance (m)} \approx \text{Range (km)} \cdot \text{Angle (mils or mrad)} . \quad (4)$$

This is an often-used unit effect, through which we find that a 1-mil angular error applied to the initial trajectory angle produces a 1-m miss distance at a 1-km range.

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### 3. How Error Sources Produce Shot Distributions

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#### 3.1 Error Types

Error sources are typically categorized in three groups:

- **Fixed Bias Errors** are inherent to a weapon system's design and stay constant for all shots, adding to the distribution's bias. The primary fixed bias error is the weapon system jump, but ballistic mismatch can also produce a significant fixed bias (also variable and random) for weapon systems that use multiple projectile types.
- **Variable Bias Errors** stay constant for shots within a given occasion but vary between occasions. Because of this, they are also sometimes referred to as "occasion-to-occasion" errors (11, 17, 24). Held et al. (24) define an occasion as "defined in terms of time, i.e., a long period (hours or days) between rounds; or in terms of significant events between rounds, such as maintenance of the weapon, large environmental changes or moving the tank to new firing positions." Variable bias errors produce an additional bias unique to the occasion that is added to the fixed bias. Some major variable bias errors are ranging error and wind error.
- **Random Errors** vary from round to round. Thus, they are sometimes referred to as "round-to-round" errors. Random errors are largely comprised of Soldier aim error and ammunition dispersion.

Some error sources are included in multiple error groups. For example, jump, which is listed as a fixed bias, also has a variable and random error component. In the ensuing descriptions, error sources that are in multiple groups will be discussed only in the first group in which they appear. It will be noted when the error source carries into subsequent groups, but it will not be discussed in depth each time.

Figure 6 shows how the error sources combine. Fixed errors bias the distribution, moving the COI to the black star. For each occasion, a variable bias is added to the fixed bias, producing the occasion's COI at each blue star. During each occasion, random errors produce the gray 1- $\sigma$  probability oval, representing the occasion's SDs.

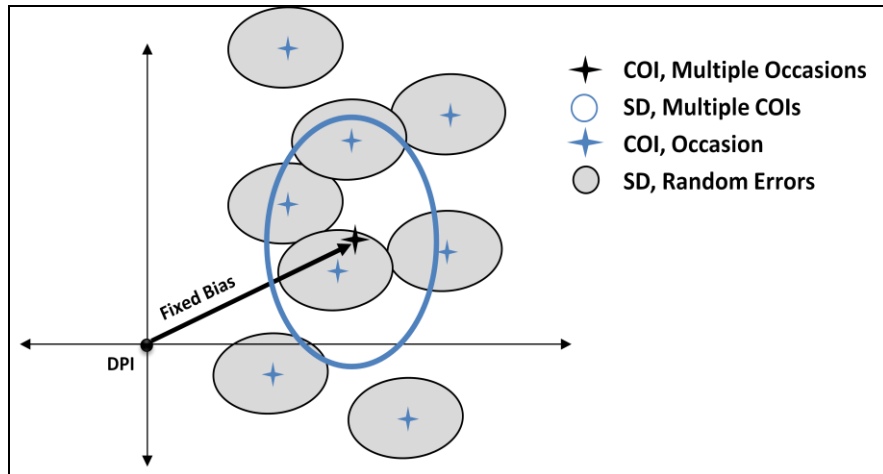


Figure 6. Error types.

Note: Target distributions displayed assuming no correlation between horizontal and vertical errors.

The goal of the shooter is to reduce the delivery error, bringing the shot distribution back toward the DPI (figure 7). This is done by predicting the fixed and variable biases and adjusting where the weapon is pointed to compensate. The gray random error probability ovals stay the same for all occasions because it is not possible for the shooter to compensate for random errors, which, by definition, are unpredictable. However, the relative size of the bias and random errors and what errors the shooter can correct for vary greatly depending on the type of weapon system and the skill of the shooter.

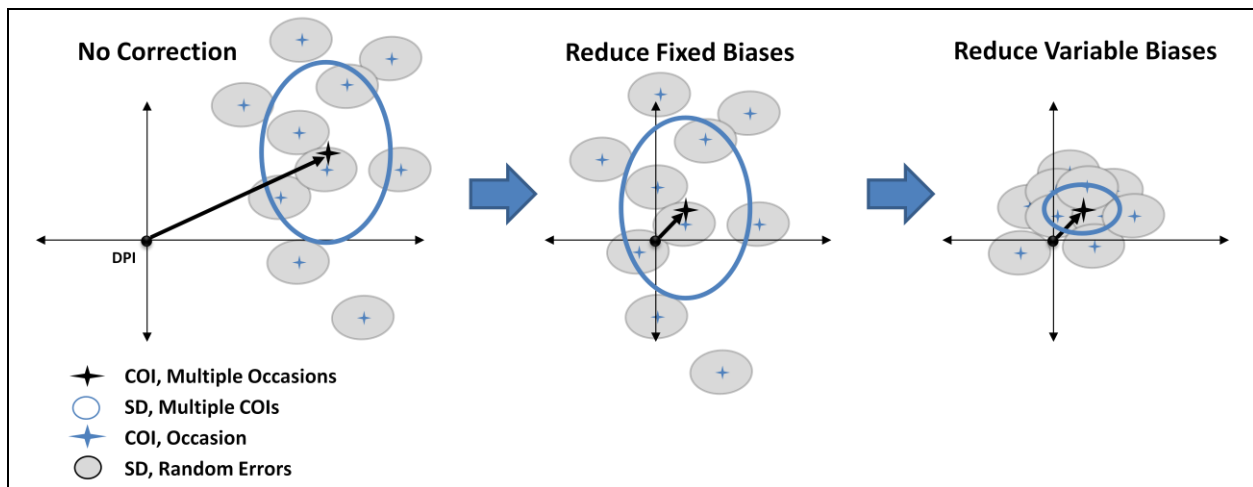


Figure 7. Reducing bias errors.

This process only characterizes the first round fired in an engagement. If the shooter is able to sense where the first projectile impacts through the use of a tracer projectile, spotter, or some other means, he can further reduce the bias of the distribution. Additional terms are needed in an error budget to quantify this reduction in offset due to sensed impacts (4).

### 3.2 Four-Stage Shooting Process

To illustrate the error types, we will now consider a four-stage process of delivering a ballistic projectile on target:

1. **Zero Weapon.** Zeroing adjusts the alignment of the weapon sight and barrel to compensate for fixed (and sometimes variable) biases in the weapon system at a given range. Typically considered separate from the zeroing procedure, a weapon is first boresighted to eliminate the parallax bias resulting from the offset of the sight from the bore (figure 8).<sup>\*</sup> This results in the line of fire (LOF), the line emerging from the barrel, intersecting the target at a different point from the line of sight (LOS), the line representing what the shooter sees. A common method of boresighting involves inserting a laser muzzle boresight device into the gun barrel, sometimes called a borelight (26), and aligning the crosshairs of the sight with the laser target projection. This only offers a true correction at the range in which the weapon is boresighted. Also, because of gravity, a projectile never travels in a straight line as assumed in the boresighting procedure and as seen in the figure. This is, however, corrected in the ensuing zeroing procedure.

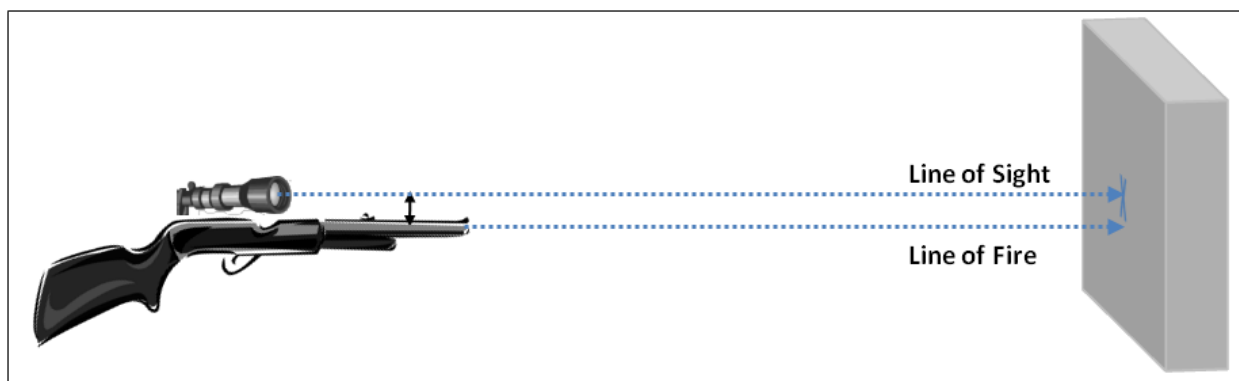


Figure 8. Parallax bias.

After boresighting, the weapon system undergoes the zeroing procedure to correct for all other fixed (and sometimes variable) biases, such as jump and gravity drop. Zeroing is done in one of two ways. In the first method, the shooter fires at a target at a known range, adjusting the sight alignment with the barrel until the shot COI is within a certain tolerance of the DPI. Alternatively, one may adjust the alignment through a fleet zero (surrogate zero), which is a known average fixed bias across a fleet of weapons at a given range. This is also called a computer correction factor (CCF), which typically applies to tank fire control systems (17, 27, 28). The vertical<sup>†</sup> angular correction applied to the trajectory,

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<sup>\*</sup> There are other sources of parallax deriving from the sight, called “optical parallax” or “focus error,” which we will discuss in section 4.2. It is typically corrected by adjusting a knob on the sight that focuses the sight to the proper range.

<sup>†</sup> In truth, the correction is applied with respect to the vector perpendicular to the weapon barrel, which may not align with the true vertical because of cant error (see section 5.4).

primarily to correct for gravity drop, is referred to as the superelevation (15, 29), while the horizontal angular correction is sometimes referred to as the azimuthal correction.

Figure 9 shows a commonly used zeroing procedure, in which one fires at a target at a shorter distance than the zeroing range and adjusts the sights until the COI is within a certain tolerance of an equivalent offset (26, 30). This process takes advantage of the fact that the trajectory intersects the LOS at two points, which are circled in blue in the figure. For many small-caliber weapons, if a 300-m zero is desired, the ideal trajectory initially intersects the LOS at a range close to 25 m. Based on the specific weapon system, trajectory analysis can be used to predict how much above or below (or to the side of) the LOS a round needs to be at 25 m to intersect the target at 300 m. This equivalent offset (red X in figure) is usually close to the LOS, and thus can fit on a small intermediate target placed at 25 m. Thus, the user fires upon the intermediate target, adjusting the weapon-sight alignment until the COI is approximate to the required offset.

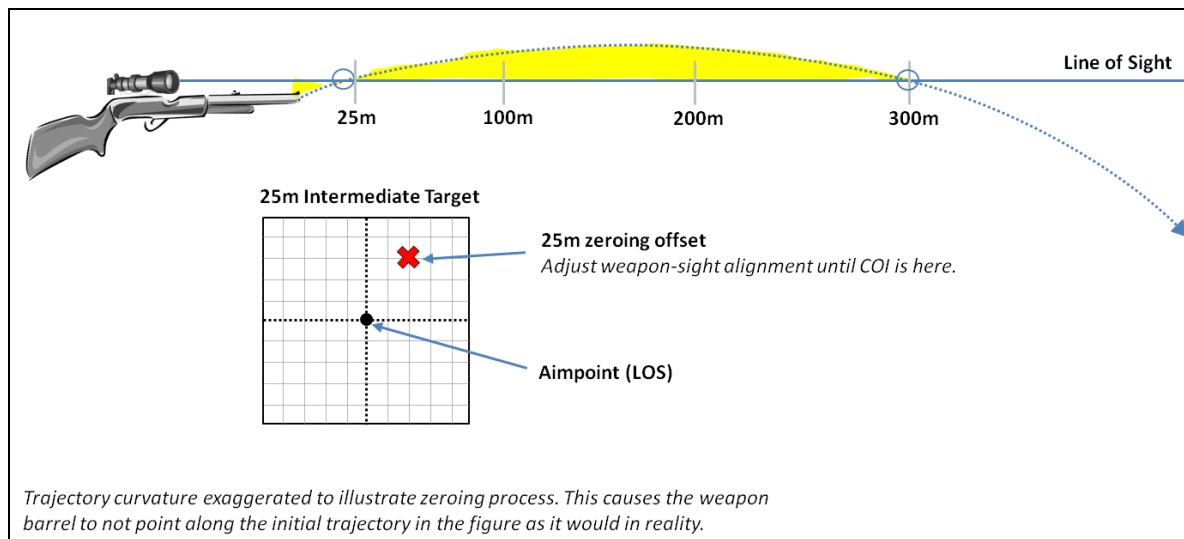


Figure 9. Zeroing procedure using an intermediate target.

Referring again to figure 9, the yellow-shaded area between the trajectory and the sight line represents the vertical miss distance as a function of range over the zeroing range. Often a shooter will rezero his weapon for each new range. However, sometimes at shorter distances, such as those shown in the figure, the shooter will only zero the weapon once and accept the inherent miss distance bias when shooting at ranges other than the zeroing range. This is referred to as a battlesight zero (26, 30), and the range at which it has been zeroed is called the battlesight range (BSR) (7). Common BSRs are 300 m for the M16A2/A3/A4 and M4 series and 250 m for the M16A1 (29). Researchers have studied the impact of a battlesight zero on  $P_h$  as a function of range (31) and have offered suggestions to improve its usage (32).

2. **Aim Weapon.** When engaging a target, the shooter may attempt to compensate for the occasion's variable bias errors, such as wind or a range other than the zeroing range. This is done either by adjusting the sight-barrel alignment or by aiming with the use of alternate ledgers marked within a sight reticle. After adjusting the weapon for the occasion, the shooter aims and pulls the trigger.
3. **Launch Event.** Upon firing the weapon, the projectile's initial trajectory angle is perturbed by gun recoil, muzzle blast, and other factors that are collectively known as jump (discussed in section 4.1). In addition, the muzzle velocity is affected by the temperature of the rounds and variations in charge mass.
4. **Flight to Target.** Once the projectile is in free flight, its motion is well-characterized by Newton's laws and resulting trajectory models (20, 33). However, small variations in drag coefficient or environmental factors such as air density or wind velocity cause the trajectory to deviate from the predicted path.

In the first two stages, the user attempts to eliminate the fixed bias and the occasion's variable biases, respectively. After factoring in aim and jump error, we arrive at the initial velocity vector as seen in figure 10A. In addition to the initial velocity vector, the total delivery error is affected by random errors that can offset the predicted trajectory, such as varying projectile characteristics (10B) and atmospheric conditions (10C).

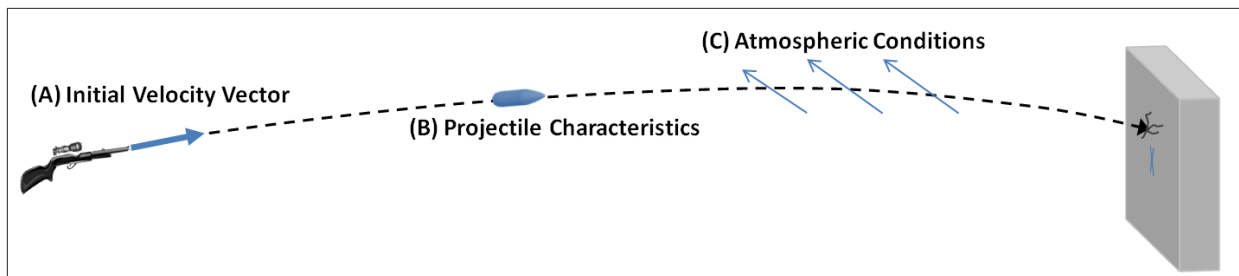


Figure 10. Factors affecting projectile trajectory.

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## 4. Fixed Bias Errors

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### 4.1 Jump

Jump quantifies the system-inherent inaccuracies within a weapon system that cause a projectile to miss the DPI. Consequently, it is measured *under very constrained conditions* that are intended to isolate the system-inherent error sources primarily associated with the launch event. It is typically expressed as the total angular deviation of the projectile as it enters free flight toward the target, which can be converted to an equivalent miss distance at a given range through trigonometry. As mentioned earlier, jump also retains variable and random error components.

In an effort to better understand jump, researchers have separated jump into smaller, measureable components, although the subdivisions and accompanying definitions have changed for different types of weapon systems and as the research community's understanding of the phenomenology has evolved (19, 34–42). In light of this, I will share two recent definitions from two different organizations.

First, as a result of years of jump-related work performed by many researchers at the U.S. Army Research Laboratory (ARL), Celmins (38) defines the following jump components, which I quote in their entirety:

- **Static-Pointing Angle:** “The angle between the gun muzzle and the line of fire (LOF) just before the trigger is pulled. This accounts for muzzle motion between the time the gun is boresighted and the shot is fired.”\*
- **Muzzle-Pointing Angle:** “The angle between the gun muzzle at the instant of shot exit and the pre-shot LOF, minus the static pointing angle. This accounts for muzzle motion after the trigger is pulled.”
- **Muzzle-Crossing Velocity:** “The angle formed by the ratio of the gun muzzle transverse velocity at the instant of shot exit to the projectile exit velocity.”
- **Center of Gravity (CG) Jump:** “Traditionally, this is broken into two components: mechanical/disengagement and sabot separation, which are described individually in this list. . . . Absolute CG jump is defined as the initial bullet trajectory angle relative to the LOF. Relative CG jump is calculated by subtracting the sum of the previous jump components from the absolute CG jump.”
  - **Mechanical Disengagement:** “Caused by the separation of the bullet from the muzzle and muzzle blast effects acting on the bullet. Measured by subtracting the sum of the previous jump components from the initial bullet trajectory angle.”
  - **Sabot Separation:** “Caused by mechanical and aerodynamic interactions between the sabot petals and the projectile. Measured by subtracting the mechanical disengagement vector from the trajectory angle after the bullet is clear of the sabots. Sabot separation was not an issue in the current test program since the small-caliber weapon systems tested did not use sabots.”
- **Aerodynamic Jump (43)<sup>†</sup>:** “Refers to the angle between the projectile's downrange trajectory and the initial trajectory angle. Aerodynamic jump is the result of the integrated effect of aerodynamic lift due to the yawing motion of the projectile. Aerodynamic jump is calculated using the initial angles and angular rates of the projectile as it enters free flight.”

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\* In this case, the LOF is defined as the boresight line before the gun muzzle (bore) has been perturbed out of alignment with the sight during testing (related to boresight retention, which we shall discuss later).

<sup>†</sup> Sometimes researchers include the effects of wind, spin drift (discussed later), and/or flow anomalies along the trajectory in the aerodynamic jump (33, 44).

- **Gravity Drop:** “Refers to the effect of gravity on the trajectory. This is directly related to, and can be determined from, the time of flight of the projectile. Converted to an angle by taking the ratio of the gravity drop to the distance from the muzzle to the target.”

Often, depending on the weapon system or applications, researchers have used different subsets of this list. For example, the static pointing angle is equivalent to the boresight retention error, which is typically carried as a separate column within an error budget. Thus, it is often not included in the total jump vector (it is vital to isolate in testing, however, so that it can be removed from the measurement). In addition, gravity drop is often subtracted from the jump vector, as it is a function of range, and it is often the goal to attain a range-independent jump angle.

Figure 11 shows a subset of the components listed by Celmins (38) and the equivalent miss distance vector contribution in the target plane. In this case, the gravity drop has been subtracted and the component vectors have been shifted to originate from the bore aim point to the DPI, because this provides a more clear representation of the miss distance. The addition of all of the component vectors produce the total jump vector, although Plostins et al. (45) have noted that the components are not always completely uncorrelated as assumed in this vector addition.

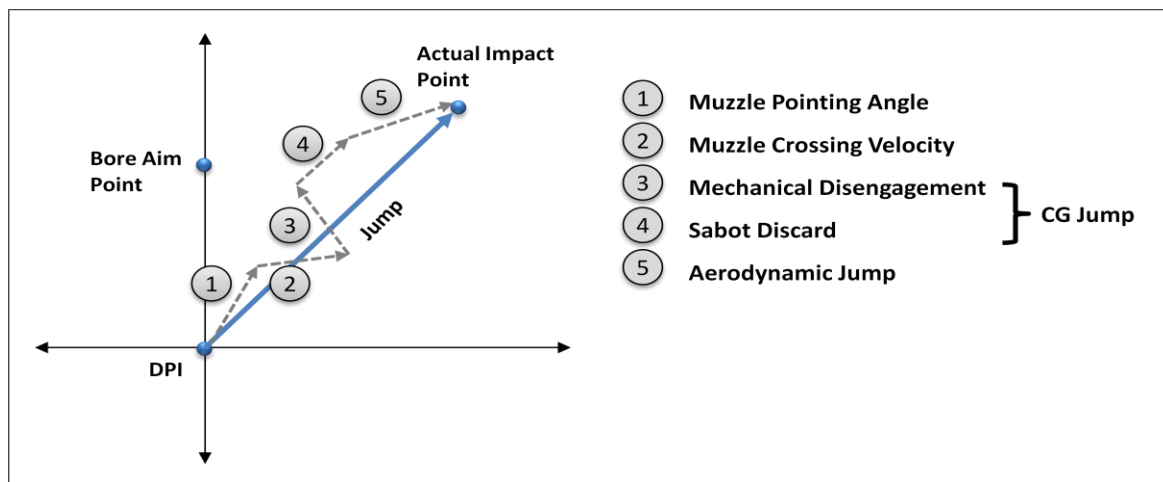


Figure 11. Example jump component breakdown.

Norman (9) of the U.S. Army Materiel System Analysis Activity (AMSAA) defines “tank jump” as composed of launch angle, curvature error, and trajectory mismatch. Launch angle has a similar meaning to the combination of the muzzle pointing angle and muzzle crossing velocity, as used at ARL. Curvature error seems to cover the CG jump, aerodynamic jump, and any other unspecified components that would increase jump. The trajectory mismatch component is the difference between the predicted fire control trajectory and the actual trajectory.\* Norman also

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\* This definition of trajectory mismatch is different from how it is defined later in this report (the trajectory difference between two projectile types).

defines “gun jump,” which is the same as tank jump except for the removal of the trajectory mismatch component.

## 4.2 Other Errors

### 4.2.1 Parallax

Because different sights can be used on a weapon system, their effects are quantified separately from the system jump. As discussed in section 3.2, a parallax bias results from the displacement of the weapon sight from the barrel, which is corrected at a given range by boresighting. In addition to this, some sights have another error, called “focus error” or “optical parallax.” The *Small Arms Integration Book* (30) defines this as “the apparent movement of the target relative to the reticle when [the] eye is moved away from the center of the eyepiece.” This occurs when the planes in which the image and reticle are projected are not coincident (44), and it is corrected by adjusting the sight focus for a given range. Because optical parallax changes the shooter’s perception of the target based on the location of the eye, it is more appropriately categorized in the variable bias and random error categories.

### 4.2.2 Spin Drift

For spin-stabilized projectiles, the projectile’s pointing vector and velocity vector do not perfectly align (figure 12, 33). The angle between the two vectors, the “yaw of repose” or “equilibrium yaw,” amounts to an angle of attack, which causes an aerodynamic lift to act sideways upon the projectile. For clockwise-spun projectiles, this causes the projectile to drift to the right. Other terms for spin drift include “bullet drift,” “gyroscopic drift,” or simply “drift.”

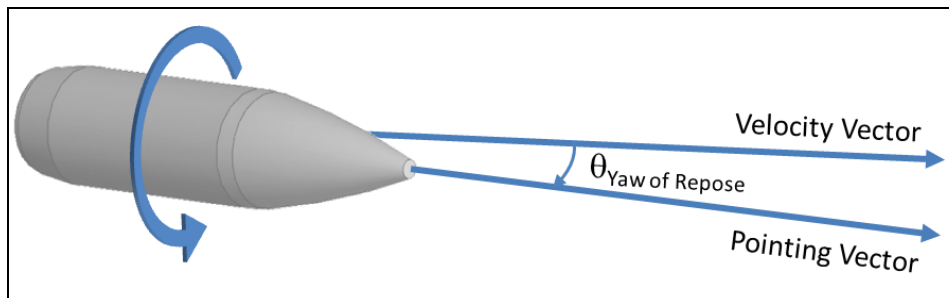


Figure 12. Spin drift.

### 4.2.3 Ballistic Mismatch

Ballistic mismatch (or the converse, ballistic match) quantifies the difference in target miss distance between target impacts of different projectile types that are shot under similar conditions.\* This may refer to projectiles that are shot from the same weapon, such as

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\* Ballistic mismatch has also been used to quantify the difference in target impacts using different gun tubes or ammunition lots.

linked ball and tracer projectiles used in machine guns (46, 47), or to projectiles that are shot from different weapons, such as a weapon system that employs both primary and spotting weapons (5, 48).

Ballistic mismatch is largely due to the different aerodynamic qualities of the different projectile types (41), but it is also due to differences in the jump (42). Thus, the ballistic mismatch has sometimes been split into both “trajectory mismatch\*” and “jump mismatch” (49). Sometimes “trajectory mismatch” is quantified exclusively in terms of the difference in gravity drop as a function of range (50). Depending on the analysis, the trajectory component of ballistic mismatch may not be necessary to carry in the error budget, as it is a function of other varying factors, such as projectile aerodynamic characteristics and muzzle velocity. Ballistic mismatch retains variable and random error components.

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## 5. Variable Bias Errors

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### 5.1 Ranging Error

Ranging error affects the superelevation applied to the trajectory, which in turn affects the miss distance in the vertical direction. There are many methods of rangefinding (7, 51), of which we shall highlight two common ones.

In the first method, the shooter measures the angle subtended by one of the target dimensions. This is done either with the naked eye or with a stadia rangefinder, which contains markings within the gun scope that measure the subtended angle. Figure 13 shows a mil-dot reticle, a popular type of a stadia rangefinder. With a mil-dot reticle, the distance between two dots subtends an angle of 1 mil. After the angle is estimated, the shooter assumes a target size and through basic trigonometry determines the range (refer to equation 5). In the figure, for example, the target width subtends approximately 1.5 mils. If we assume that the target width in the figure is 0.5 m, we estimate the range as

$$Range (m) \approx \frac{Target Size (m) \cdot 1000}{Angle (mils)} = \frac{0.5 m \cdot 1000}{1.5 mils} = 333 m . \quad (5)$$

A second method of rangefinding works by illuminating the target with a laser and determining the time it takes for the laser to transmit to the target and back.<sup>†</sup> Knowing the speed of light, we can determine the distance. While inherently very accurate, laser rangefinders can exhibit poor performance in bad atmospheric conditions or when the user does not properly designate the target, leading to underspill or overspill onto unintended objects (7, 53).

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\* Recall Norman’s definition of trajectory match with respect to jump, which has a different meaning.

<sup>†</sup> This is the “time of flight” method of laser rangefinding. Other methods of laser rangefinding also exist, such as the phase shift method or interferometry (52).

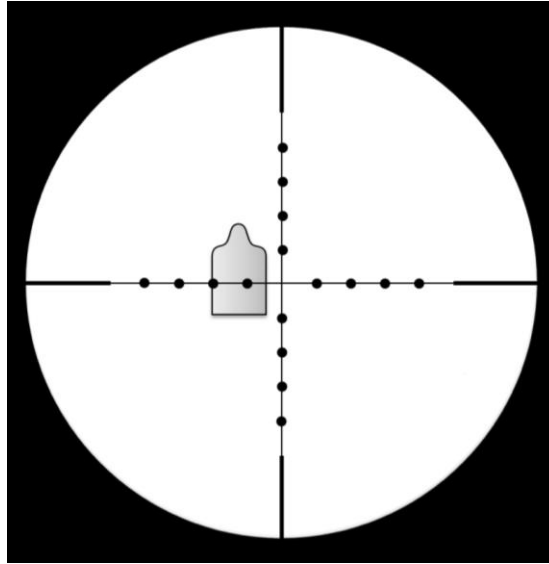


Figure 13. Mil-dot reticle.

## 5.2 Wind Error

Wind is typically resolved into components parallel and perpendicular to the flight path. The parallel component is called “range wind,” and the perpendicular component is called “crosswind,” which affect the miss distance in the vertical and horizontal directions, respectively. Crosswind generally has a much larger effect than range wind on flat-fire ballistic trajectories, and it is common to not consider the effects of range wind in an error budget (54). Wind is often compensated for by predicting the crosswind and adjusting the weapon sight. The error in prediction, especially considering that the wind velocity is described by a “wind field” that may be very different at different parts of the trajectory, and in correction (i.e., the resolution of the sight’s windage adjustment) together produce the wind error for the engagement. Wind error also retains a random component for every shot fired, often termed “wind gustiness.”

## 5.3 Components Carried Down From Fixed Bias Group

In addition to their fixed components, jump and ballistic mismatch produce a unique variable bias for each occasion.

## 5.4 Other Errors

### 5.4.1 Zeroing Error

When zeroing is accomplished through firing, the statistically small number of rounds fired at the target and the tolerance demanded of the process (i.e., when the shot pattern is considered “close enough” to the DPI and the process concludes) produce errors. For fleet zeroing, the difference between the fleet average bias and the bias of the weapon being used produces an

error. In addition, for both firing and fleet-zeroing processes, if atmospheric conditions, target range, or muzzle velocity are inaccurately measured during the zeroing process, the user's ability to properly correct for new conditions on different occasions is degraded.

#### 5.4.2 Boresight Retention Error

Between the time the weapon is boresighted and when it is fired, the alignment between bore and sight can be unknowingly perturbed because of weapon handling, vibrations, etc. This change in alignment produces an error in the projectile's initial trajectory angle from the weapon.

#### 5.4.3 Muzzle Velocity Variation

Muzzle velocity variation (MVV) results from variations in projectile temperature and propellant mass. MVV affects the range of the projectile and leads to miss distance in the vertical direction. Because this error is largely temperature-dependent, it could also be defined as "temperature error" (5) (in this case, variations in propellant mass leading to round-to-round MVV would likely be included in bullet dispersion—see section 6.2).

#### 5.4.4 Air Density Error

Lower air density decreases the drag force experienced by the projectile, causing the projectile to travel farther and to impact higher in the target plane.

#### 5.4.5 Cant Error

Cant is the angle between the local "upwards" direction of the gun tube and the vertical direction as defined by gravity. Cant results from the gun tube being rolled about its long axis, which can occur because of uneven terrain or if the shooter simply fails to hold the gun level (figure 14). For small angles of cant, it leads to miss distance mainly in the horizontal direction. Cant error is sometimes referred to as "trunnion roll" when referring to tank accuracy.

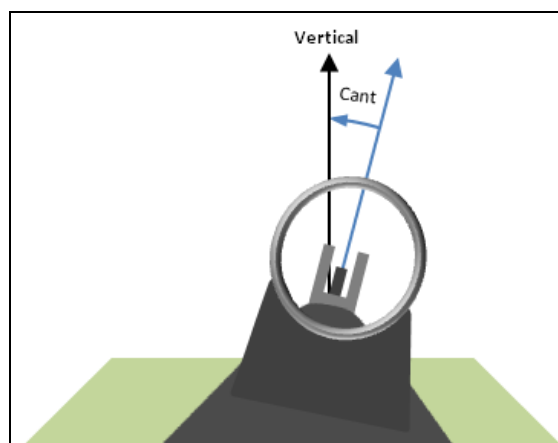


Figure 14. Cant angle.

#### 5.4.6 Weapon-Target Altitude Error

An altitude difference between shooter and target that is not accounted for in the fire control solution will contribute to the miss distance in the vertical direction.

#### 5.4.7 Optical Path Bending

Optical path bending occurs because of refractive index changes along the LOS (optical path) and is characterized by Snell's law, which characterizes the deflection of light as it enters a medium with a differing refractive index. Refractive index of a gas is dependent on pressure and temperature, and thus phenomena such as turbulence and thermal gradients change the refractive index and cause an object's location to appear displaced as viewed by an observer (i.e., a mirage). In addition, this can cause the object to appear to be moving when it is not. Optical path bending introduces an error into the weapon pointing angle.

#### 5.4.8 Fire Control Error

This error quantifies the error in both predicting the correct firing angle necessary to hit the DPI and implementing these corrections in adjusting the angle of the weapon/sights. Norman (9) refers to this as being composed of at least "computation error" and "implementation error," although implementation error includes aim/lay error, which would be carried as a separate random error source for small-caliber systems. Many of the errors involved in correcting bias errors, such as ranging error or wind error, are typically quantified in their own respective sections. The accuracy of the predicted ballistic trajectory, however, would typically be included in this category.

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## 6. Random Errors

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### 6.1 Aim Error

Aim error is caused by the gunner's inability to line up the sight with the desired aim point at the time of fire. Aim error is sometimes referred to as "pointing error" (11) or "lay error" (4, 9, 46). There are many factors that determine its magnitude:

- **Human vs. robotic gunner.** For a human gunner, the levels of stress, training, and time to aim have a major effect on the aim error (7, 55).
- **Target impact observation.** The ability to observe projectile impacts and make corrections throughout an engagement generally improves the hitting performance of subsequent rounds (4, 5).\*

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\* In cases when random errors are much larger than bias errors, implementing corrections from shot to shot can actually be counterproductive—a "chasing your tail" scenario (56). In this case, corrections should be delayed until a larger number of impacts have been observed.

- **Burst fire vs. single shot fire.** Some researchers have included burst-to-burst variation as a separate bias above round-to-round errors in the error budget (46, 47, 55, 57).
- **Static vs. moving target (53).** Additional error budget terms are necessary to characterize moving targets, which are covered in Norman's report (9).
- **Sight resolution and magnification.**

## 6.2 Ammunition Dispersion

Ammunition dispersion (or “bullet dispersion”) is caused by variations in the manufacturing process, which lead to variations between rounds in mass, shape, propellant, etc. Ammunition dispersion is typically quantified for a round/weapon combination, such as the M855 bullet from the M16A2 rifle (7). The variability in aerodynamic qualities is sometimes considered in a separate category as the muzzle retardation variability (MRV) (20) or the variation in ballistic coefficient. Sometimes variations in muzzle velocity from round to round are considered in a separate category from ammunition dispersion (see section 6.3).

As an important side note, the terms “round-to-round dispersion” (RRD) or merely “dispersion” are sometimes used synonymously with ammunition dispersion (2, 4, 9, 46). However, these terms can also be used to refer to the *total* random error, which includes additional errors, such as aim error and wind gustiness (7). Thus, care must be taken when using these terms.

## 6.3 Components Carried Down From Fixed/Variable Bias Groups

- Jump
- Muzzle velocity variation
- Wind gustiness
- Optical path bending (affecting aim error)
- Ballistic mismatch

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## 7. Summary and Recommended Reading

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In this report, I described the main sources of delivery error for direct-fire ballistic projectiles. Error sources and the resulting delivery errors are typically assumed to behave according to normal distributions, which can be specified through a mean and SD. There are three major categories of error sources: fixed bias errors, variable bias errors, and random errors. For a weapon system, fixed bias errors always remain constant, while random errors always change. Variable bias errors stay constant within an engagement but change between engagements. For a

distribution, biases shift the distribution's COI off of the desired impact point, while changing error sources modify the distribution's SDs.

Throughout the report, I have attempted to reference reports relevant to each error source discussed. However, I would like to highlight a few reports that have been most helpful if the reader seeks to extend his knowledge of the subject in a more broad sense. Grubbs (8) provides a helpful explanation of different ways to characterize the precision of shot distributions.

Brodkin's (3) explanation of the factors affecting tank accuracy and dispersion is considered by many to be one of the first major works on the topic, and many subsequent studies point back to it as a foundational source. In addition to outlining many of the error sources, it also defines different types of conditions in which the errors may be measured, such as "User Service Test" and "Quasi-Combat." Weaver's (7, 25) reports not only describe the different error sources in detail but also quantify many of their values for small-caliber weapon systems. Lastly, Norman (9) systematically defines the error sources affecting tank accuracy and precision, some of which were not described in this report, and also compares how terms are used in the United States as opposed to in the United Kingdom.

By understanding the error sources and the way they influence delivery error, one is able to estimate the total delivery error and the corresponding first-round  $P_h$  against a target. A helpful follow-on to this report would be a survey of the values attributed by researchers to the different error sources, which would help to lay the framework for a comprehensive error budget for direct-fire munitions.

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